A small 'basic number facts' study in an intermediate school setting

by Des Rainey

Baseline tests in a high-decile intermediate school setting showed students to be at low levels of accuracy and fluency with both multiplication and division basic facts. In multiplication the average accurate facts per minute score (AFPM) was 12, equating to a very slow 5 seconds per fact. Students scored at 10 AFPM in division, 6 seconds a fact. A three month intervention with a set of basic fact teaching methods brought very substantial gains, namely from 12 to 18 AFPM on average in multiplication and from 10 to 15 AFPM in division. Students' awareness of their improvement week by week and their overall very positive attitude to the programme, even enjoyment, accompanied this success.

Performance at baseline was very varied, ranging from two students who scored above 30 AFPM to a third of all students with AFPM scores lower than five. The preponderance of low scores suggested that the school might be carrying a particularly heavy share of students with educational problems. It raised the wider question of how many students are coming into intermediate schooling untrained in ability to recall basic facts and needing remedial help.

Remarkably increased variance on re-test after 3 months appeared due to the relatively high gains made by students who were relatively advanced to begin with. It was hypothesized that this trend might reverse or at least ameliorate when students who were singularly deficient in certain knowledge and skills to begin with 'found their feet', as some of them showed signs of doing. Importance was seen, where students made little or no progress, in ascertaining the reasons and making instructional adjustments to suit.

From inspection of sets of facts that students had difficulty with, it was of interest and concern that at baseline, only about a third of these Year 6 and 7 students knew *all* of the individual facts within the 4, 6, 7, 8 and 9 times tables while approximately two thirds knew all of the facts within the 2, 3 and 5 times tables. It appeared obvious that teachers and students at the start of a school year should know precisely which facts or sets of facts (tables) needed attention – likewise that students maintain some sort of record as a guide to managing and monitoring their basic fact learning through the year. *AFPM Tests* and *Knowledge-Scale Basic Fact Progress Charts* were examples of tests and charts designed for these purposes.

Background

The participating school was a small private school whose Principal held firmly to the belief that in mathematics every student needs to develop basic knowledge and skills. A school newsletter to parents relayed certain views, also promoted by OECD, on the need for students to do foundation maths very well and indicating the likely future outcomes for those with the skills and for those without. As the Principal said, 'At the level of basic maths it is absolutely key to have structure, persistence, repetition… hard work with good strategies… and all sorts of things that are not entertaining or fashionable. We need to keep insisting all of the children are able to do those basics to enable them to think and succeed at higher levels.'

As a practical measure, the school opted to trial a basic facts learning programme developed by the present author and known from previous research to be highly effective (Rainey 2012). Known to students as 'Kiwi Maths' the programme provided short concentrated systematic practice opportunities in a distinctive form. Key features included the challenge of competing against oneself, error-free learning, the development of accuracy before speed, and immediacy of reinforcement. Goal-setting and self-monitoring contributed to student empowerment. The enjoyment which the programme typically generates was perhaps the most important factor of all – along with teacher support.

Parents were informed of what the school sought to achieve and their support, ideas and suggestions were invited.

All of the children in two classes engaged in the study – 20 in all – of which 80 per cent were male.

The research course

The study took place in the last 3 months of the school year. For the school it provided the opportunity to evaluate *Kiwi Math* methods and help confirm that their proposed wider use in the coming school year would be warranted. The researcher saw it in part as an opportunity to undertake a small formal study of basic fact learning at Intermediate school level to see what could be learned. At all events, following baseline testing the school ran *Kiwi Maths* 'from the book' as it saw fit.

The tests used for baseline purposes and review were the *AFPM Tests of Basic Facts* (Rainey 2011). The main measure they provided was *Accurate Facts per Minute* (AFPM), a combined measure of accuracy and speed which relates to fluency, competence and productivity. The tests also enabled basic number fact *Knowledge-Scale* ratings (Rainey 2014) to be made. These, from 0 to 10, take out the speed element and measure only knowledge.

The intervention

The Principal noted: 'Three times a week - in my science class - we did practice and timed events for an average of 10-15 minutes. The students quickly got into a routine and clearly enjoyed what they were doing. They did their own timing and recording of exercises. We started at the tables 5 to 9 level and this extra challenge seemed to work for the children. Without exception there was improvement with what we did in class. We notified parents of what we were doing and asked for lots of practice at home. We emailed to tell them what we were focusing on in each section. I also emailed with general comments on improvement to help keep the momentum going.'

Results

The outcomes are shown first in terms of *Accurate Facts per Minute (AFPM)* and then in respect of factual knowledge as from *Knowledge-Scale* ratings.

Because by curricular standards all students were of an age where they might be expected to know their basic facts, Year 7 and Year 8 AFPM results were combined. They are shown In Table 1.

Table 1: Year 7 and 8 multiplication and division accurate facts per minute

Year 7 and 8 Multiplication AFPM		
	Baseline	Final
N	20	20
Median	9	13
Mean	12.3	17.5
SD	11.9	16.6

Average 3-month gain: 5.3	Effect Size : 0 .37

Year 7 and 8 Division AFPM		
	Baseline	Final
N	19	19
Median	6	9
Mean	10.3	14.8
SD	12.7	16.6

Average 3-month gain: 4.5	Effect Size : 0.31

In multiplication the average gain (the difference between means) was 5.3 Accurate Facts Per Minute. Considering that it came not from a whole year's work but from just three months, the Effect Size of .37 indicated very substantial improvement.

Division results were similar to those for multiplication. The mean gain was 4.5 AFPM and the Effect Size, while a little lower at .31, was still exceptionally good for a 3-month intervention.

However, in both multiplication and division there was marked variation between students. The best performing student in multiplication at baseline, for example, moved from 46 AFPM to 64 or to less than a second a fact. Other students made good or satisfactory progress but a substantial number (close to half) made little or no progress or even went backwards.

Basic Fact Knowledge

AFPM test sheets make clear precisely which facts in a table are answered correctly or incorrectly or are omitted. For research purposes one may derive other information such as the percentage of tables fully known by each student, as in Table 2.

Table 2. The percentage of tables fully known.

Multiplication								
% fully known	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Baseline	59	59	36	64	41	36	32	36
3-month re-test:	73	55	55	50	55	45	41	55
Division								
Baseline	55	41	36	45	36	36	27	41
3-month re-test:	55	41	41	32	32	41	41	36

The school chose to work on the 6, 7, 8 and 9 multiplication and division set. At baseline the items in these four tables were wholly known on average by only 36% of students. This changed to 49% post intervention. There was only a small gain, from 55% to 58%, in the tables not specifically worked on. Broadly speaking, in division there were no clear patterns of change.

Student feedback

On a 10-point self-rating scale on how competent they considered themselves with multiplication facts the average student rating was 4 for before and 7 for after the intervention. The division rating was 3 for before and 7 for afterwards. At the least, this suggests very positive overall student engagement in the project.

Students were also asked to frankly answer: 1) How helpful was *Kiwi Maths?* 2) Were you pleased with the amount of effort you put in? 3) Did you have any difficulties along the way? 4) 'Is there anything else you would like to say about *Kiwi Maths* or about learning basic facts?'

Responses to the 'helpfulness' item could be categorised as indicating 'very helpful' in 11 cases, helpful in 7, of some help in 1, and not helpful in 1. It did again seem clear that in general the students felt positive about the programme and, significantly it is thought, *enjoyed* it.

Individual comments included:

- It was very helpful but I am not too good on 7s, 8s 9s. I am still slow.
- I think a bit with 6, 7 and 8 but my mum taught me a trick for 9. I think it should be compulsory that each school must teach its students this so they learn confidence.
- It was very helpful because I sort of forgot my 7 basic facts from when I was in year 6. It was amazing fun. I loved it.

- It was helpful a lot because I know a lot of times and division and I didn't know a lot but now I do. It helps me a lot with my maths.
- Would recommend to anyone if they struggle. [!]
- Extremely helpful. This IS a difficuilty (for me). I was pleased with my effort but not my outcome.
- It was really helpful because I didn't have time to do it at home but now at school I can do it, so it was great. I just want to say it is a great idea and has help me very much.
- Not so helpful but i did learn a bit and I did get good times but not so much, especially the 9s. Difficulties? A lot. Some of the times tables I have never come across before and I never lerned the 6, 7, 8, 9 at my old school and I have no idea how to divide. im not a big fan of maths as you can tell!
- it hoped me wit dufisun a lot
- It was quite helpful as we did it twice a week.
- I found it helpful for increase my time table and division speed.

In passing, degree of parental interest and involvement was not measured but would be worth specific attention in future studies.

Discussion and recommendations

The major finding was that overall, the student group made remarkable progress in its short time on the *Kiwi Maths* memorisation of basic facts programme. It is confirmation of the effectiveness found in earlier research and notably in the Fairburn study of 2011.

Almost as important was the marked variance found at baseline and even greater variance on re-test. Performance at baseline ranged from two students above 30 AFPM to seven with AFPM scores less than 5. But how could one explain the even greater variance found after 3 months? It is thought to relate to relatively high gains being made by students who were relatively advanced to begin with. The correlations were 0.36 in the case of multiplication facts and 0.62 in division. It is possible, that as advanced students progress towards their upper limit and plateau out, and as those who are initially deficient in certain respects begin to 'find their feet', the observed trend in these correlations might be reversed.

What else can we conclude? A degree of advancement in basic fact knowledge is a clearly stated pre-requisite for engagement in the *Kiwi Maths* programme so the fact that students who were relatively advanced at the start tended to be those that did well is perhaps not surprising. Conversely, the students who were at a very low level to start with may have benefited more from attention to preparatory work and/or from engaging in a simpler and less abstract programme. In the *Number Stick* programme, for example, students use manipulatives and go through phases of building a specific table and playing a game for that table to start it into memory before engaging in direct memory work on the same table.

The foregoing thoughts are seen now to fit with the observation by Hattie and Yates (2014) that '*Learning proceeds quickly and efficiently when what is new builds directly upon what is already secured.'* The students in the present study who did not do well appear not to have had a sufficient foundation to build on. In future similar studies one sees the need to clearly identify the knowledge and skills which students present and make sure they are offered an instructional course – or pace - to suit. Success must be judged accordingly and individually. For one student it may be a large gain in 'accurate facts per minute', for another it may be a giant step forward to get all the facts of the two-times table into memory.

At all events, when students are found to be making little or no progress after several weeks under a particular teaching approach one should try and find the reasons so that in light of these they continue, perhaps at a slower pace, or change to something else. Questions which should be in mind in individual review include:

- Can physical/organic and psychiatric problems be ruled out?
- Is there a problem relating to anxiety or confidence?
- Is there a general or specific attitude problem or apathy?
- Are family support and circumstances relevant?
- Does the student's understanding and competence or level of development in basic fact knowledge provide a platform to move forward from? Might it be better or necessary to go back to work at a lower developmental level or might it suffice to keep to a slower initial pace and try and build confidence? (In the present study the teaching decision for all students to work on the tables 6 to 9 would have better seen some of them obtain confidence and security with the tables 2 to 5.)
- Is there *gross* lack of understanding? Is there rank confusion? Is the very concept of division not clear to the student? If so, that is where to start.
- Is a proposed approach to basic fact learning suitable for a particular student? For example, in the earliest stages of working on tables the more physical and concrete *Number Stick* approach (Rainey 2010) would be suitable when *Kiwi Maths* would not.

Standards

Ideally, the current results could also be evaluated by how they relate to official standards – but *there are no officially recognised standards or norms.*

Until the Ministry sees their necessity and provides them, we can at least show how findings relate to other research results. We relate them to informal standards sanctioned by parent groups, as described in the author's earlier research. Two seconds per fact (30 AFPM) was seen as an acceptable end-target and standard by parents. Three seconds per fact (20 AFPM) was considered satisfactory and 4 seconds (15 AFPM) as unsatisfactory and requiring special help. Children who were still lower (at the time when a good standard was normally reached) as in serious remedial need.

As so defined, the standards achieved by the students in the current study are shown in Table 3.

Table 3. Classification of students by AFPM levels/standard

AFPM	Year 7 & 8 Multiplication		Year 7 & 8 Division	
	baseline	3-month	baseline	3-month
60+		1		1
55-59				
50-54				
45-49	1	1	1	
40-44				1
35-39		1		
30-34	1	1	1	1
25-29	1	1	1	1
20-24	2	1		
15-19	2	2	2	1
10-14	3	3	1	4
5-9	3	5	4	6
0-4	7	4	9	4

One sees clearly here the upward progression and the likelihood that a number of students will achieve a good or at least satisfactory standard. They should stay on Kiwi Maths.

Students in the 15-19 range are of concern, and more so are those in the lower shaded areas.

A significant number of students are at a gravely low level and should be individually assessed as to why this is so and how their needs may be best served. Methods *other than Kiwi Maths* methods are advised for the most basic learning and understanding of multiplication and division. The strength of *Kiwi Maths* is in 'finishing the job' and helping students reach automatic recall and/or a level of personal excellence. The Number Stick approach to tables (Rainey 2010) may suit students who need a simpler and more physical and concrete programme. In the most severe cases a grounding in *Number Family Maths* (Rainey 2002) might be considered. It is a programme which focuses on the most basic understanding and learning of number relationships. At all events, a careful study of scores and patterns of response in *AFPM* testing should substantially guide teachers as to the instructional path that will help a particular child.

A general recommendation from this study was that where a school does not already have a way for students to monitor their progress in basic fact learning it should introduce one. Preferably student progress charts would first display students' knowledge at the start of a school year, as shown by some form of test. It should also show progress through the year and provide an end-of-year test result. It should help students set and revise goals - and in that respect could be both motivating and reinforcing. It should also of course provide information to teachers and parents. The *Knowledge-Scale™ Basic Fact Progress Charts* (Rainey 2011) illustrated below attends to all these aspects. They can be used independently of whatever teaching methods and tests are being used but have the advantage of linking smoothly with *AFPM* tests and *Kiwi Maths* methods.

The Multiplication Tables I Know!

My basic fact progress through the year (notional example)

BASED+	TEST BASED+	Term 1	Term 2	Term 3	Term 4	TEST
10 times		1	1	1	1	1
9 times					1	1
8 times					1	1
7 times				1	1	
6 times			1	1	1	1
5 times		1	1	1	1	1
4 times		1	1	1	1	1
3 times	1	1	1	1	1	1
2 times	1	1	1	1	1	1
1 times	1	1	1	1	1	1
MULTIPLICATION Scale Score™	M3	M6	M7	M8	M10	M9

Score 1 for each family-of-facts marked as 'known'. (See notes on credit for 'one times' and 'ten times'.)

*'Know' means I can answer any item in a table and can show you what it means. Try me!

From such a chart one sees that the student started with knowledge of tables up to 3, learned tables 4 and 5 in Term 1, gradually mastered other tables and ended by knowing all but one of the 10 tables to criterion. It is hypothesised that keeping a progress chart such as this may be found to have actual motivating and reinforcing benefits for many students.

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